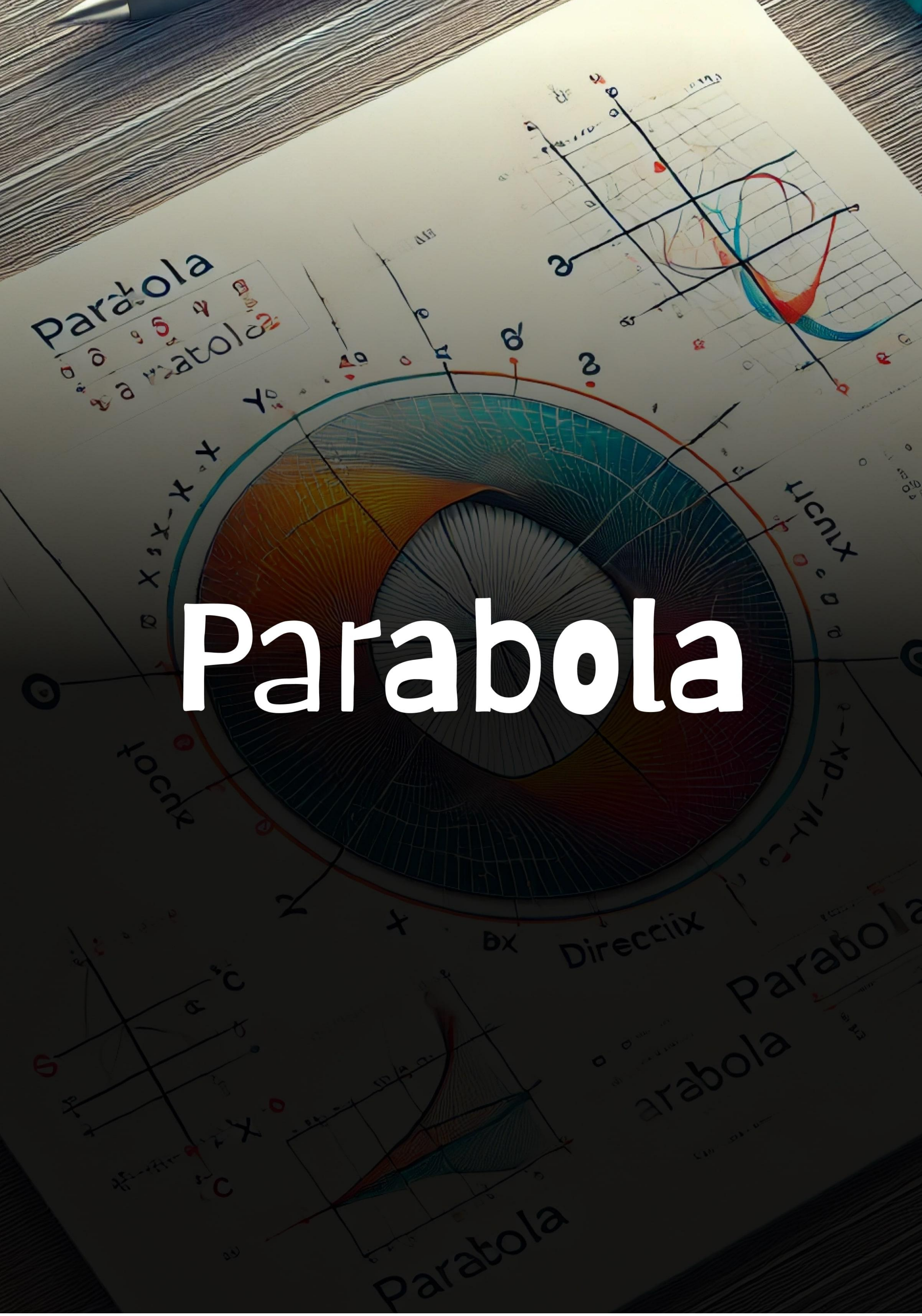
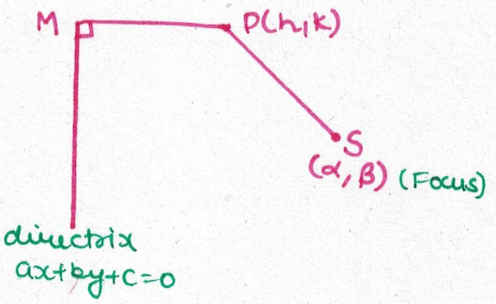


Parabola

parabola

DEFINATION The locus of a point which moves in such a way that the distance of the point and from a fixed point and from a fixed line is always equal.



$e = \frac{SP}{PM}$

For parabola, $e = 1$

$SP = PM$

The fixed pt. is focus (s) and the fixed line is directrix. Locus is known as **Parabola**.

Eccentricity (e) is the ratio of SP and PM.

i.e. $e = \frac{SP}{PM}$

$$\boxed{(x-\alpha)^2 + (y-\beta)^2 = \left|\frac{(ax+bx+c)^2}{a^2+b^2}\right|}$$

Ques: Find the equation of a Parabola whose S(2,3) and directrix, $x+y+1=0$

Sol: $(x-2)^2 + (y-3)^2 = \frac{(x+y+1)^2}{2}$

$\Rightarrow 2(x^2+4-4x) + 2(y^2+9-6y) = x^2+y^2+1+2x+2y+2xy$

$\Rightarrow x^2+y^2-2xy-10x-14y+25=0$

> The second degree general equation → $ax^2+by^2+2hxy+2gx+2fy+c=0$ represents a parabola when,
>
> $\Delta \neq 0$ and $h^2 = ab$
>
> $\Delta = abc + 2fgh - af^2 - bg^2 - ch^2$

Ques: $S(a,0)$, directrix → $x+a=0$

Sol: $(x-a)^2 + y^2 = (x+a)^2$

$y^2 = (x+a)^2 - (x-a)^2$

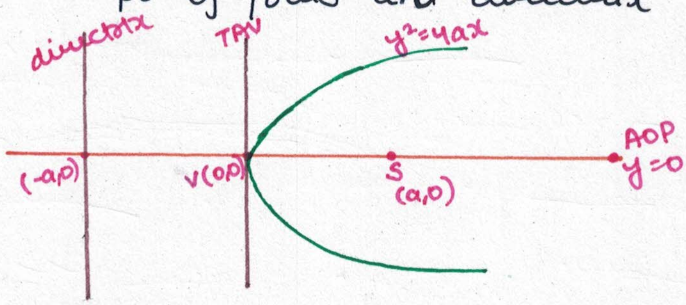
Parabola - $y^2 = 4ax$

TERMINOLOGY

- **Axis of Parabola (AOP)** - The line of symmetry which contain focus of the parabola is known as AOP.
- **Tangent at Vertex (TAV)** - A line which is $\perp r$ (perpendicular) to AOP and passing through the vertex of parabola.
- **Vertex (V)** - The POI of parabola and AOP is Vertex
- **Focus (S)** - The fixed pt. which lies on AOP is known as focus
- **Directrix (D)** - The fixed line which is $\perp r$ to AOP and is of the same distance from vertex to focus in opposite direction

$VS = VD$

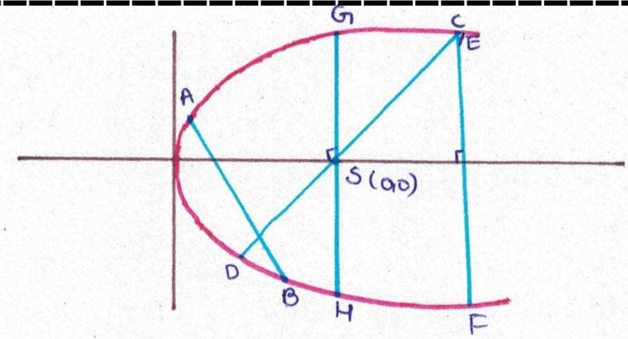
vertex is the mid pt. of focus and directrix



- **Chord** - A line which cuts the parabola at 2 distinct points is known as chord
- **Focal chord** - A chord which passes through focus
- **Double Ordinate** - A line which is $\perp r$ to AOP and cuts parabola at 2 distinct points is known as double ordinate.
- **Latus Rectum** - Double ordinate passing through focus is known as LR

$\boxed{\text{Length of Latus rectum} = 4a}$

Basically, Latus rectum is also a chord, focal chord and doube ordinate to parabola.



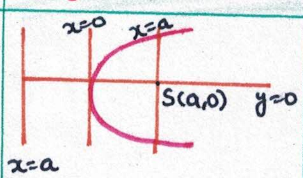
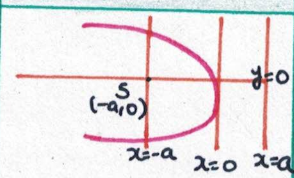
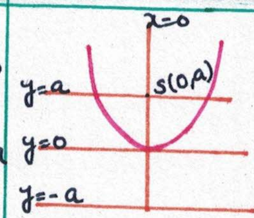
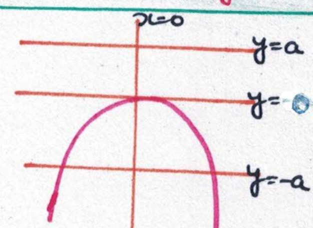
AB, CD, EF, GH are chords

CD, GH → Focal chords

EF, GH → Double ordinate

GH → Latus rectum

STANDARD PARABOLA

Memorize!	$y^2 = 4ax$	$y^2 = -4ax$	$x^2 = 4ay$	$x^2 = -4ay$
Graph				
Focus	$(a, 0)$	$(-a, 0)$	$(0, a)$	$(0, -a)$
Axis of Parabola	$y=0$	$y=0$	$x=0$	$x=0$
TAV	$x=0$	$x=0$	$y=0$	$y=0$
Directrix	$x=-a$	$x=a$	$y=-a$	$y=a$
LR	$x=a$	$x=-a$	$y=a$	$y=-a$
Length of LR	$4a$	$4a$	$4a$	$4a$

Ques: $x^2 = 16y$

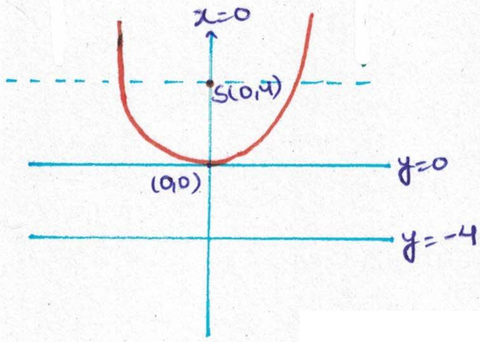
Sol: $V = (0,0)$

$S = (0,4)$

axis → $x=0$

TAV = $y=0$

LR = $y=4$



Ques: $(y-4)^2 = 8(x+7)$

Sol: $V = (-7,4)$

$a = 2$ $\quad S = (a, 0)$

$S(-5, 4)$

LR = 8

$x+7 = 2$

$x = -5$

$y = 4$

Latus rectum – $x = a$ $\quad x+7 = 2$ $\quad \Rightarrow x = -5$

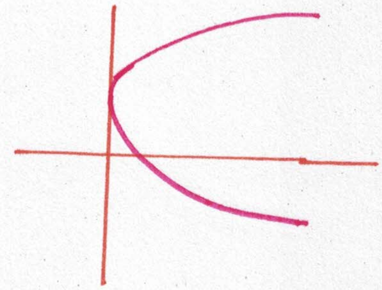
Directrix – $x = -a$ $\quad x+7 = -2$ $\quad \Rightarrow x = -7$

TAV – $x = 0$ $\quad \Rightarrow x = -7$

Ques: $y^2 - 6x - 6y + 9 = 0$

sol: $(y^2 - 2(3y) + (3)^2) = 6x$

$(y-3)^2 = 6x \quad = 4\left(\frac{6}{4}\right)x \qquad a = \frac{3}{2}$



$V = (0,0) = (0,3)$

$S = (a,0) = \left(\frac{3}{2}, 3\right)$

AOP = $y = 0 \Rightarrow y = 3$

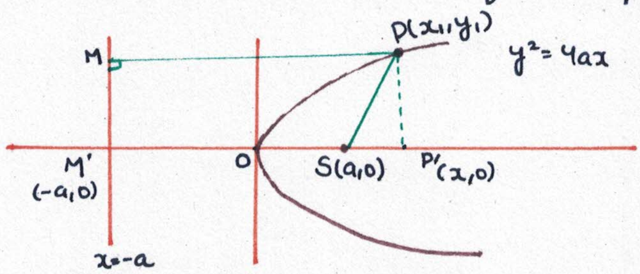
TAV = $x = 0 \Rightarrow x = 0$

Directrix $x = -a \Rightarrow x = -3/2$

LR = $x = a \Rightarrow x = 3/2$

FOCAL LENGTH

The distance of a point on Parabola from focus is known as **Focal length**.



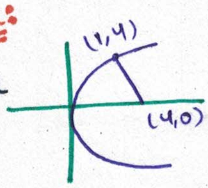
$PS = PM \Rightarrow PS = P'M' = OP' + OM'$

$PS = |x_1| + a$

> Focal length $= |x| + a$
>
> $\Rightarrow$ (mod of co-ordinate x of given point) $+ a$

Example:

$y^2 = 16x$



Focal length $= |1| + 4 = 5$

or we can also apply distance formula and find the focal length.

$d = \sqrt{(4-1)^2 + (4-0)^2} = \sqrt{16+9} = 5$

> NOTE: If the parabola is vertical, then focal length will be – $|y_1| + a$

Position of a Point w.r.t. Parabola

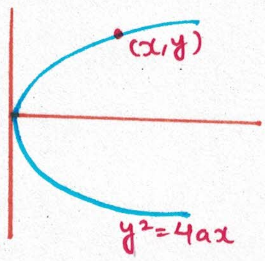
If we wish to find the position of a pt. w.r.t. parabola $y^2 = 4ax$ then first satisfy the pt (x_1, y_1) in the equation of parabola i.e. $y_1^2 - 4ax_1$

If $y_1^2 - 4ax_1 > 0$ Outside

$y_1^2 - 4ax_1 = 0$ On

$y_1^2 - 4ax_1 < 0$ Inside

Parametric Coordinates



$y^2 = 4ax$

$\frac{y}{2a} = \frac{2x}{y} = t$

t is a parameter

$y = 2at$

$2x = yt = 2at^2$

$x = at^2$

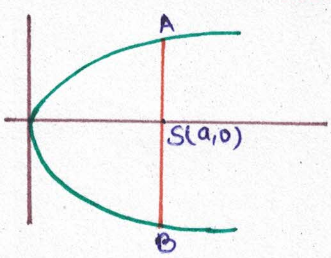
$x = at^2,\ y = 2at$ } Parametric equation of parabola

Parametric co-ordinates are-

$\boxed{(at^2, 2at)}$ $t \in R$

Ques: If $A(t_1)$ and $B(t_2)$ are 2 pts. on the parabola $y^2 = 4ax$, then find the relation between t_1 and t_2 if AB is a focal chord.

Sol:



$A = (at_1^2, 2at_1)$ $B = (at_2^2, 2at_2)$

$\Rightarrow \frac{2at_2 - 2at_1}{at_2^2 - at_1^2} = \frac{0 - 2at_1}{a - at_1^2}$

$\Rightarrow \frac{t_2 - t_1}{t_2^2 - t_1^2} = \frac{-t_1}{1 - t_1^2}$

$\Rightarrow \frac{t_2 - t_1}{(t_2 - t_1)(t_2 + t_1)} = \frac{-t_1}{1 - t_1^2}$

$\Rightarrow t_1^2 - 1 = t_1 t_2 + t_1^2$

$\Rightarrow \boxed{t_1 t_2 = -1}$

If a parabola has a focal chord then, the coordinates of its end pts on parabola are $(at^2, 2at)$ and $(a/t^2, -2a/t)$

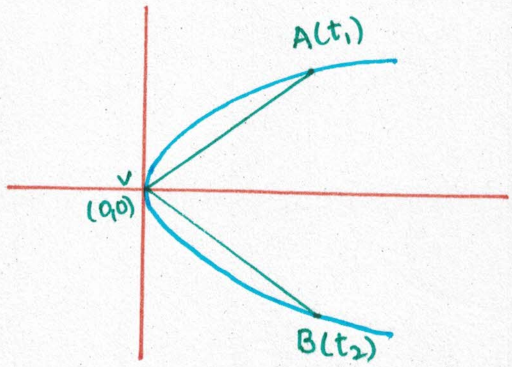
Remember → $t_1 t_2 = -1$ → For focal chord

Ques: If the line joining $A(t_1)$ and $B(t_2)$ subtend right angle at the vertex of the parabola. $y^2 = 4ax$. Then find relation between t_1 and t_2

Sol:

$$m_{AV} = \frac{2at_1}{at_1^2}$$

$$m_{BV} = \frac{2at_2}{at_2^2}$$



$\Rightarrow \quad m_{AV}\, m_{BV} = -1$

$\Rightarrow \quad \dfrac{2at_1}{at_1^2} \cdot \dfrac{2at_2}{at_2^2} = -1$

$\Rightarrow \quad \boxed{-4 = t_1 t_2}$

Note: Focal chord cannot subtend 90° at the vertex and a chord which subtend 90°, at the vertex cannot be a focal chord.

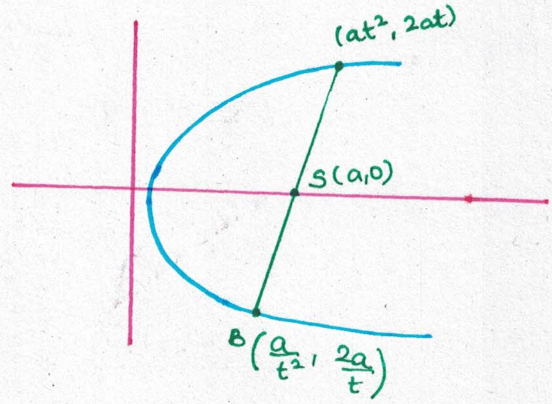
Ques: For the parabola, $y^2 = 4ax$, find the length of the focal chord in parametric form and also prove that the length of latus rectum is $4a$.

Sol: Length focal chord $= AS + BS$

$$= at^2 + a + \frac{a}{t^2} + a$$

$$= a\left(t^2 + \frac{1}{t^2} + 2\right)$$

$$= a\left(t + \frac{1}{t}\right)^2$$



For AB to be latus rectum

$at^2 = a \quad \Rightarrow t = \pm 1 \qquad t = 1$ acceptable

length of latus rectum $= a(1+1)^2 = 4a$

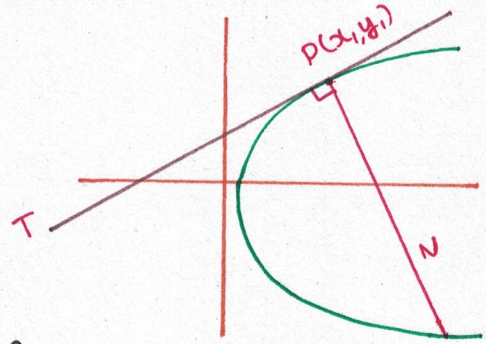
The length of focal chord to a parabola with parameter 't' is-

$$\left| a\left(t + \frac{1}{t}\right)^2 \right|$$

TANGENTS AND NORMALS

TANGENT AT THE POINT

*** POINT FORM:** Equation of tangent to the parabola $y^2 = 4ax$ at $P(x_1, y_1)$, then it will be given as $T=0$



$YY_1 - 2a(x+x_1) = 0$

$$\boxed{YY_1 = 2a(x+x_1)}$$

*** EQUATION OF NORMAL:**

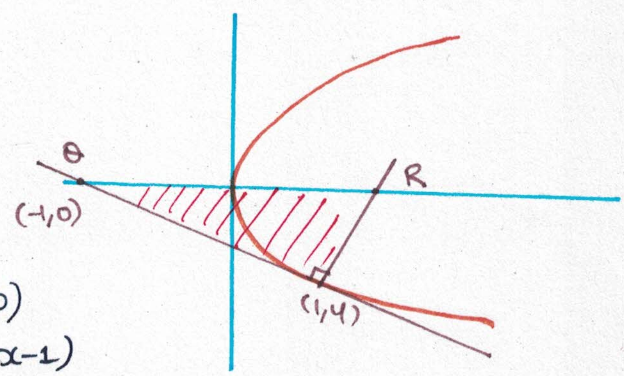
Slope of tangent at $P(x_1, y_1) = \frac{2a}{y_1}$

$m_1 = -y_1/2a$

Equation of normal

$$\boxed{Y - Y_1 = \left(\frac{-Y_1}{2a}\right)(x - x_1)}$$

Ques: Find the area of the Δ formed by AOP of the parabola, $y^2 = 16x$ with the tangent and normal at $(1, 4)$



Sol: Eq^n of tangent-

$y(-4) = 8(x+1)$

$-4y = 8x+8$

$8x + 4y + 8 = 0$

Eq^n of AOP-

$y = 0$, $x = -1$, $Q(-1, 0)$

Eq^n of normal $(y+4) = \frac{-4}{2(4)}(x-1)$

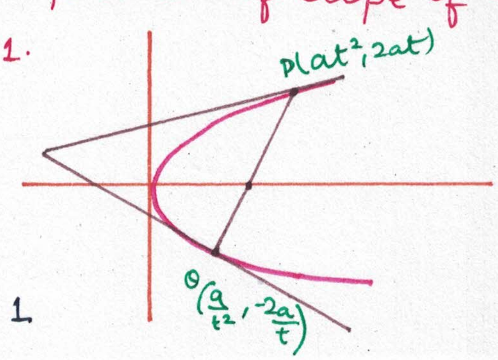
$2y + 8 = -x + 1$

$2y = -x - 7$ $\quad\quad y = 0$

$R(9, 0)$ $\quad\quad x = 9$ $\quad\quad x = -7$

area $= \frac{1}{2} \times B \times 4 = \frac{1}{2} \times 20 \times 2 = 20$ units

Ques: A chord is drawn inside the parabola $y^2 = 4ax$ which is passing through focus. Prove that the product of slope of the tangents drawn the extremities is -1.



Sol: Tangent at P-

$y(2at) = 2a(x + at^2)$

at Q,

$y\left(-\frac{a2}{t}\right) = 2a\left(x + \frac{a}{t^2}\right)$

$\Rightarrow m_1 \times m_2 = 2a/2at \times 2at/2a = -1$

Points To Remember

1. If we draw 2 tangents at the extremities of focal chord, then they are always mutually perpendicular

2. Slope of the tangent at any pt. P $(at^2, 2at)$ is $(1/t)$

3. The slope of normal at any pt. P $(at^2, 2at)$ will be $(-t)$.

Tangent And Normal In Parametric Form

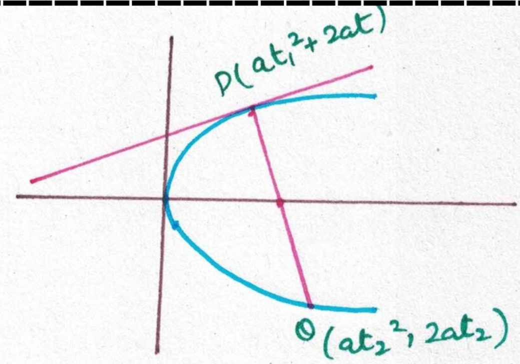
If we wish to find the equation of tangent and normal at any pt. P on $y^2 = 4ax$ parametrically, then substitute $x_1 = at^2$ and $y_1 = 2at$

Hence,
$$yy_1 = 2a(x + x_1)$$
$$y(2at) = 2a(x + at^2)$$
$$\boxed{yt = x + at^2} \quad \text{\} Equation of Tangent}$$

$$(y - y_1) = \frac{-y_1}{2a}(x - x_1)$$
$$(y - 2at) = -\frac{2at}{2a}(x - at^2)$$
$$(y - 2at) = -t(x - at^2)$$
$$y = 2at - xt + at^3$$
$$\boxed{y = -xt + 2at + at^3} \quad \text{\} Equation of Normal}$$

Ques: If we draw normal to the parabola $y^2 = 4ax$ which meets the parabola again at a pt., then prove that this normal cannot pass through the focus of the parabola. Co-ordinate of P is (t_1) and Q is (t_2) where P is a pt. from which normal is drawn and meets the parabola again at Q. Find relation between t_1 and t_2.

Sol:
$$y = -xt + 2at + at^3$$
$$S(a, 0)$$
$$y = -at + 2at + at^3$$
$$y = at + at^3$$
The normal does not pass through focus.



$\Rightarrow \dfrac{2at_2 - 2at_1}{at_2^2 - at_1^2} = \dfrac{y - 2at_1}{x - at_1^2}$

$\Rightarrow 2at_1 = -at_1^3 + 2at_1 + at_1^3$

$\Rightarrow 2at_2 = -at_2^3 + 2at_2 + at_2^3$

$\Rightarrow 2t_1 = -t_1^3 + 2t_1 + t_1^3$

$\Rightarrow 2at_2x - 2at_1x - 2a^2t_1^2t_2 + 2a^2t_1^3$
$= yat_2^2 - 2a^2t_1t_2^2 - yat_1^2 + 2a^2t_1$

$\Rightarrow 2t_2x - 2t_1x - 2t_1^2t_2 + 2t_1^3 - yt_2^2 + 2t_1t_2^2 - yt_1^2 - 2t_1^3 = 0$

$\Rightarrow -y(t_1^2 + t_2^2) + 2x(t_2 - t_1)$

$\Rightarrow y = -x(2t_2 - 2t_1)$

$\Rightarrow 2at_2 = -at_1t_2^2 + 2at_1 + at_1^3$

$\Rightarrow at_2(2 + t_1t_2) = at_1(2 + t_1^2)$

$m_{PQ} = \dfrac{2at_2 - 2at_1}{at_2^2 - at_1^2} = \dfrac{2(t_2 - t_1)}{(t_2 - t_1)(t_2 + t_1)}$

$m_{PQ} = \dfrac{2}{t_2 + t_1}$

$\dfrac{2}{t_2 + t_1} = -t_1$

$\boxed{t_2 = -t_1 - \dfrac{2}{t_1}}$

$t_1^2 + t_1t_2 = -2 \qquad t_1^2 = -1$

$t_1t_2 = -1$

Ques: If normal at $A(t_1)$ and $B(t_2)$ meets the parabola again at $C(t_3)$, then find relation between t_1 and t_2.

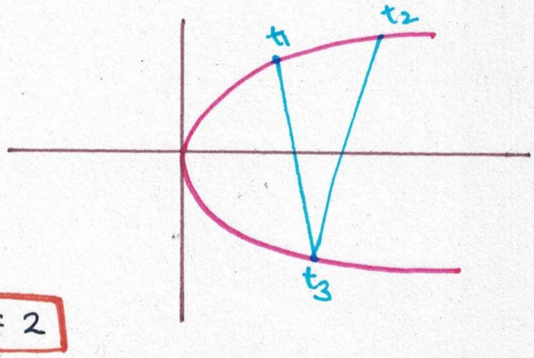
Sol: $y^2 = 4ax$

$y = -xt_1 + 2at_1 + at_1^3$

$y = -xt_2 + 2at_2 + at_2^3$

$t_3 = -t_1 - \dfrac{2}{t_1} = -t_2 - \dfrac{2}{t_2}$

$\dfrac{-t_1^2 - 2}{t_1} = \dfrac{-t_2^2 - 2}{t_2} \Rightarrow \boxed{t_1t_2 = 2}$



TANGENT AND NORMAL IN SLOPE FORM

Let $y = mx + c$ is tangent to parabola $y^2 = 4ax$

So, put the value of y in parabola.

$$(mx+c)^2 = 4ax$$

$$\Rightarrow \quad m^2x^2 + c^2 + 2mxc - 4ax = 0$$

$$\Rightarrow \quad (m^2)x^2 + (2mc - 4a)x + c^2 = 0$$

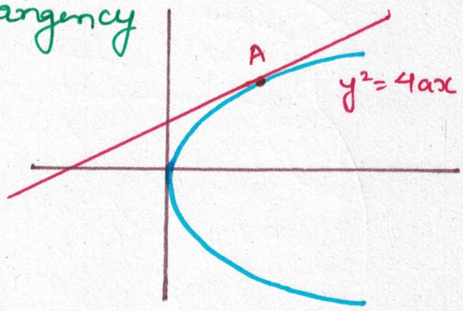
$\because$ The tangent will cut parabola at only 1 pt. = D of eq. should be 0.

$$D = 0 \quad \Rightarrow \quad 4(mc - 2a)^2 - 4m^2c^2 = 0$$

$$m^2c^2 + 4a^2 - 4mac - m^2c^2 = 0$$

$$4a^2 = 4mac$$

$$\boxed{c = \frac{a}{m}} \Big\} \text{ condition for tangency}$$



or eq. of tangent is -

$$\boxed{y = mx + \frac{a}{m}}$$

$m \rightarrow$ slope of tangent

From the above eq^n it is clear that we cannot draw two parallel tangents to a given parabola.

$$m = \frac{1}{t} \qquad P(at^2, 2at)$$

$$\boxed{P = \left(\frac{a}{m^2}, \frac{2a}{m}\right)} \Big\} \text{ co-ordinates of pt. P in slope form}$$

As we know,

eq^n of normal to parabola $y^2 = 4ax$ in parametric form is -

$$\boxed{Y = -xt + 2at + at^3}$$

slope of normal $m = -t$

So, eq^n of normal in slope form is -

$$\boxed{y = mx - 2am - am^3}$$

$m \rightarrow$ slope of normal

POI of normal to parabola

$$\boxed{P = (am^2, -2am)}$$

★ If parabola is shifted eg. $(y-\beta)^2 = 4a(x-\alpha)^2$, then equation of tangent becomes-

$$\boxed{(y-\beta) = m(x-\alpha) + \frac{a}{m}}$$

Ques: Find the eqⁿ of tangent to the parabola $y^2 = 4ax$ which is also the tangent to the circle $x^2+y^2=1$.

Sol:

$y = mx + c$ $\qquad\qquad$ $y = mx \pm a\sqrt{1+m^2}$

$y = mx + \frac{a}{m}$ $\quad (c = a/m)$ $\qquad$ $c = a\sqrt{1+m^2}$

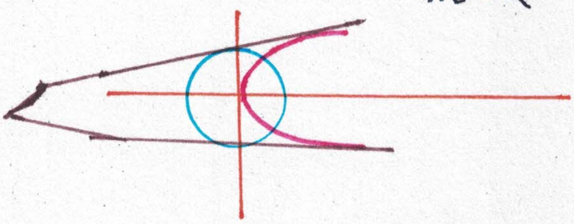
$\frac{a}{m} = a\sqrt{1+m^2}$

$y = \sqrt{\frac{-1 \pm \sqrt{1+4a^2}}{2}}\,x + \frac{a}{m}$

$m^2 = \alpha$

$\alpha^2 + \alpha - a^2 = 0$

$\alpha = -\frac{1 \pm \sqrt{1+4a^2}}{2} = \frac{-1 \pm \sqrt{5}}{2}$



TANGENT & NORMAL

QUICK REVISION

Parabola → $y^2 = 4ax$

① Point Form

Tangent : $yy_1 = 2a(x+x_1)$

Normal : $(y-y_1) = \left(\frac{-y_1}{2a}\right)(x-x_1)$

② Parametric Form

Tangent : $yt = x + at^2$

Normal : $y = -xt + 2at + at^3$

③ Slope Form

Tangent : $y = mx + a/m$

Normal : $y = mx - 2am - am^3$

TANGENT & NORMAL FROM A POINT

If the point P lies outside the parabola, then we can draw 2 tangents to the parabola.

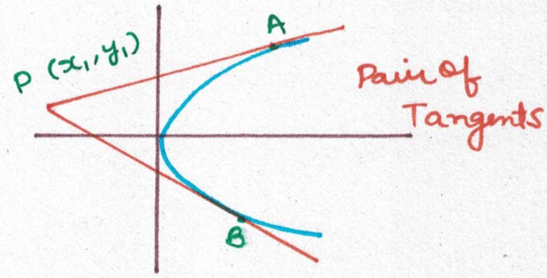
Combined Form:

$SS_1 = T^2$

$S = y^2 - 4ax$ $\qquad\qquad$ $S_1 = y_1^2 - 4ax_1$

$T = yy_1 - 2a(x+x_1)$

where (x_1, y_1) is a pt. lying outside to parabola $y^2 = 4ax$



Separate Form:

Substitute the pt. $P(x_1, y_1)$ in the tangent in slope form and make a quadratic in 'm'. We will get 2 values of m and 2 equations of tangent.

1- Let $y = mx + \frac{a}{m}$ is tangent to $y^2 = 4ax$

2- Substitute the pt. (x_1, y_1) in $y = mx + \frac{a}{m}$ i.e. $y_1 = mx_1 + \frac{a}{m}$

3- We get a quadratic equation in m. Value of m will provide 2 tangents.

Ques: $y^2 = 32x$. Find the equation of tangent to this parabola from $(0,4)$.

Sol: $4 = \frac{8}{m}$ $\quad m = 2$ $\quad y = 2x + \frac{8}{2}$

Tangent $\rightarrow y = 2x + 4$ $\qquad$ other tangent $x = 0$

Ques: $y^2 = 4x$. Two tangents are drawn from $(0,10)$. Find the area of the Δ formed by tangents with x-axis.

Sol:

$10 = \frac{1}{m}$, $m = \frac{1}{10}$

$y = \frac{1}{10}x + 10$

$10y = x + 100$

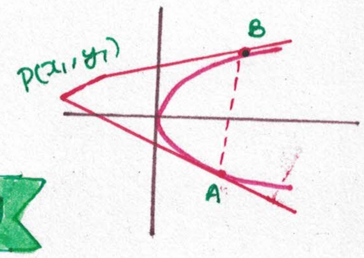
$x = 0, \quad y = 10$

Area $= \frac{1}{2} \times 100 \times 100 = 5000$

EQUATION OF CHORD OF CONTACT

If we draw 2 tangents outside a pt. P to the parabola $y^2 = 4ax$, then the line joining the pt. of contact is known as chord of cont-act and the equation of chord of contact will be given by $T = 0$.

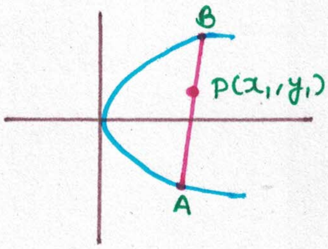
i.e. $\boxed{yy_1 - 2a(x+x_1) = 0}$



EQUATION OF CHORD WHOSE MID POINT IS GIVEN

If $P(x_1, y_1)$ is the mid pt. of the chord AB, then eqn of chord AB will be $T = S_1$

i.e. $\boxed{yy_1 - 2a(x+x_1) = y_1^2 - 4ax_1}$



Ques: If we draw 2 tangents to parabola $y^2 = 4ax$ from any pt. on the directrix, then-

i) Prove that angle between the tangents is $90°$

ii) The chord of contact is a focal chord

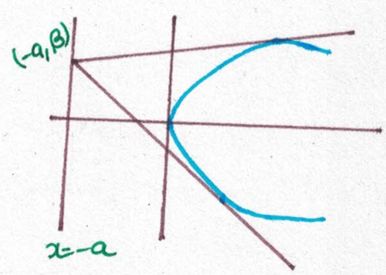
iii) In case of parabola, the directrix behaves as a directrix circle.

Sol: i) $y = mx + \frac{a}{m}$ pt. $(-a, \beta)$ $\beta = m(-a) + \frac{a}{m}$

$am^2 + m\beta - a = 0$

$m_1 m_2 = -\frac{a}{a} = 1$

Hence the tangents are perpendicular



ii) pt. $(-a, \beta)$

$y(\beta) = 2a(x-a)$ $S = (a, 0)$

It passes through $(a, 0)$

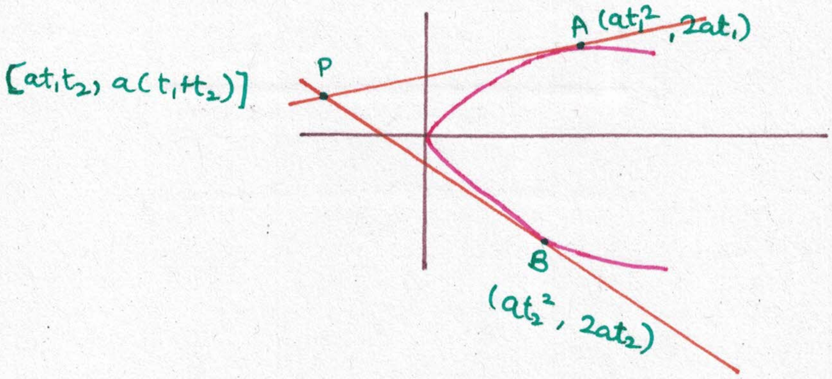
Hence it is a focal chord

iii) In case of parabola, all the perpendicular tangents intersects at directrix. In other words, all the pts. on directrix will draw perpendicular tangents to parabola.

* In parabola, if we draw tangents at the extremities of the focal chord, then their pt. of intersection will lie on directrix

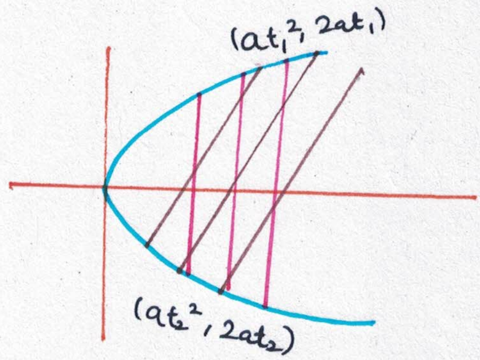
** If we draw tangents to parabola $y^2 = 4ax$ at $A(t_1)$ and $B(t_2)$ then the co-ordinates of POI of the tangents are -

$$\underbrace{at_1t_2}_{\text{GM of x-coordinate}},\ \underbrace{a(t_1+t_2)}_{\text{AM of y-co-ordinate}}$$



Ques: Find the locus of mid pt. of the chords which are parallel.

Sol: Equation of the chord is (h, k) is



$$yk - 2a(x+h) = k^2 - 4ah$$

$$2ax - yk + 2ah - 4ah + k^2 = 0$$

$$m = \frac{2a}{k} \qquad k = \frac{2a}{m}$$

$$\text{locus} - \quad y = \frac{2a}{m}$$

This line is called diameter

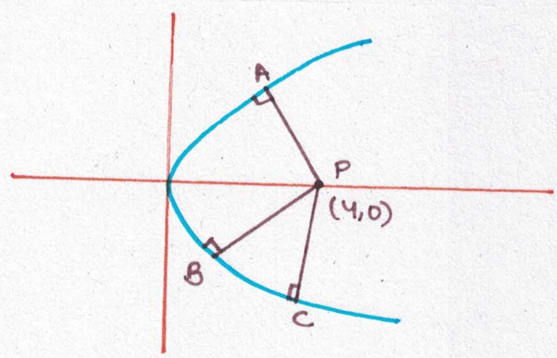
Ques: Find the condition for h when normals drawn to the parabola $y^2 = 4ax$ from $(h, 0)$ are-

(i) Only one

(ii) 3 normals all are coincident on AOP

(iii) Are 3 distinct normals.

Sol: Let eq^n of normal is



$$y = mx - 2am - am^3$$

it satisfies $P(h, 0)$

$$0 = hm - 2am - am^3$$

$$m(am^2 + 2a - h) = 0$$

$$am^2 + (2a - h) = 0$$

Case I: $2a - h > 0$

$2a > h$ One normal i.e. AOP

Case II: $2a - h = 0$, $m = 0$, $m = 0$

$h = 2a$

3 coincidents normals on AOP

Case III: $2a - h < 0$ $\quad 2a < h$

3 distinct normals

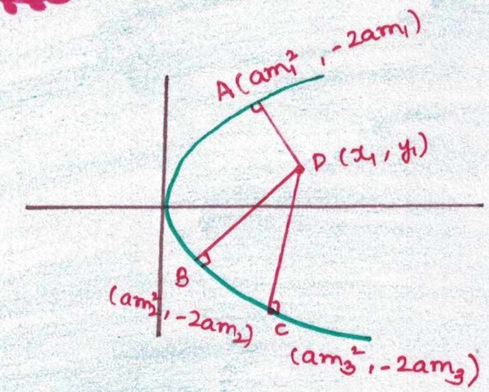
NOTE

$y = mx - 2am - am^3$

$y_1 = mx_1 - 2am - am^3$

$am^3 + 2am - mx_1 + y_1 = 0$

$am^3 + (2a - x_1)m + y_1 = 0$



Let this cubic equation has 3 roots-

m_1, m_2, m_3 then,

$$\boxed{\begin{aligned} m_1 + m_2 + m_3 &= 0 \\ m_1m_2 + m_2m_3 + m_3m_1 &= \frac{2a - x_1}{a} \\ m_1m_2m_3 &= -y_1/a \end{aligned}}$$

- A, B and C are said to be co-normal pts.
- The sum of the ordinates of all three co-normal pts. is zero
- From a pt. $P(x_1, y_1)$ either we can draw one normal or at max. 3 normals.

TRANSFORMATION RULE

Parabola		Tangent
⇒ $y^2 = 4ax$	$E(x, y, a) = 0$	$y = m_T x + a/m_T$
Normal -	$y = m_n x - 2am_n - am_n^3$	$(at^2, 2at)$
⇒ $y^2 = -4ax$	$E(x, y, -a) = 0$	$y = m_T x - \frac{a}{m_T}$
	$y = m_n x + 2am_n + am_n^3$	$(-at^2, -2at)$
⇒ $x^2 = 4ay$	$E(y, x, a) = 0$	$x = my + a/m$
	$x = my - 2am - am^3$	$(2at, at^2)$
⇒ $x^2 = -4ay$	$E(y, x, -a) = 0$	$x = my - a/m$
	$x = my + 2am + am^3$	$(-2at, -at^2)$

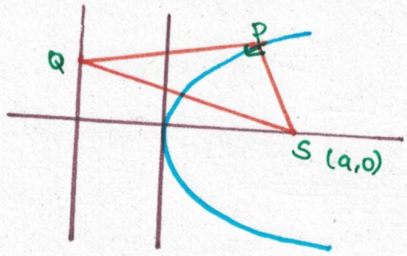
PROPERTIES OF PARABOLA

1. If we draw tangents from any point on the directrix to the parabola, than angle between them will be $\pi/2$.

2. The chord of contact of 2 tangents drawn from any pt. on the directrix will be a focal chord.

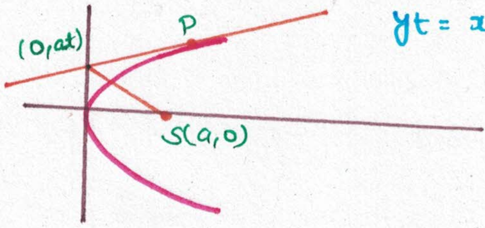
3. If we draw 2 tangents at the extremities of the focal chord of parabola, then their POI will be on directrix

4. If we take the portion of contact and directrix, then this portion subtend an angle of $\pi/2$ at focus.



5. No normal can pass through focus of parabola except AOP.

6. If we draw perpendicular to any tangent from S, then the foot of the perpendicular lies on TAV

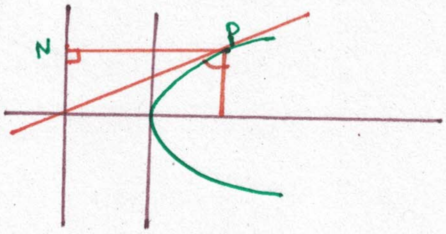


$yt = x + at^2$

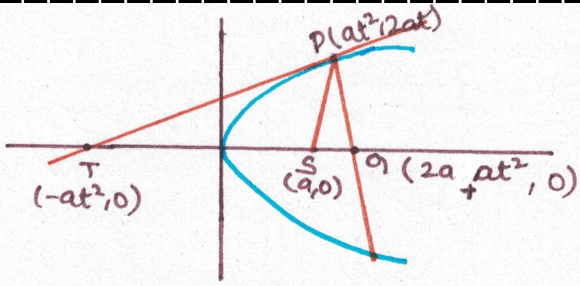
$$\frac{x-a}{t} = \frac{y-0}{-t} = \frac{-(a+at^2)}{1+t^2}$$

$x = 0, \; y = at$

7. The tangent at any pt. P is the angle bisector of focal chord from P and perpendicular line to the directrix through P.

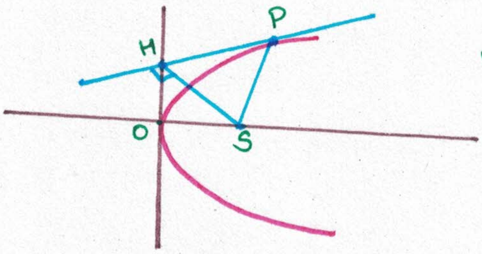


8. If we draw tangent and normal at the pt. P on parabola which cuts the AOP at T and G, then ST = SG = SP



ST = SG = SP = $a + at^2$

9 If we draw any tangent at a pt. P on parabola which meets TAV at H then $SH^2 = OS \cdot SP$



$SH^2 = OS \cdot SP$

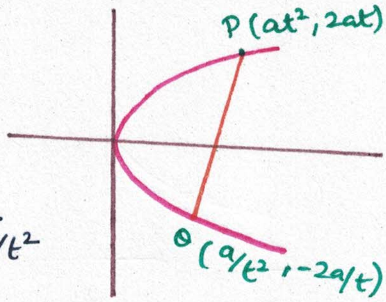
10 The length of semi latus rectum of parabola is the harmonic mean of the portion of the focal chord between focus and parabola.

$PS = a + at^2$

$QS = a + a/t^2$

$\frac{1}{PS} + \frac{1}{QS} = \frac{1}{a+at^2} + \frac{1}{a+a/t^2}$

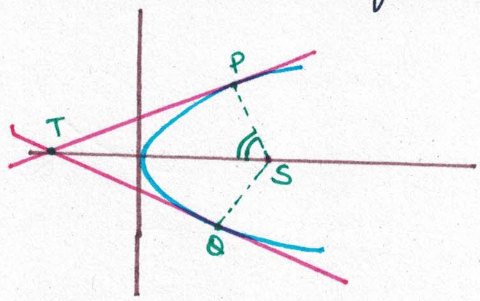
$= \frac{1}{a}$



$\frac{2(AS)(SQ)}{AS+SQ} = 2a$

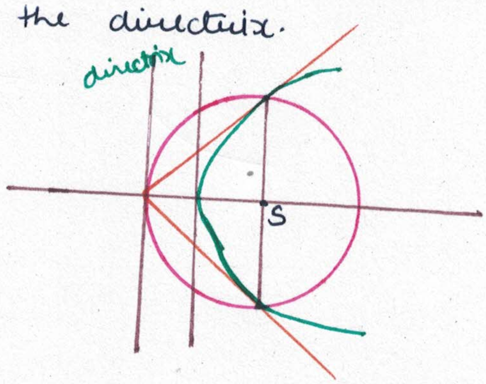
$\frac{1}{a} = \frac{1}{SP} + \frac{1}{SQ}$

11 If we draw two tangents at P and Q which meets at the point T, then TP and TQ subtends, equal angle at the focus.

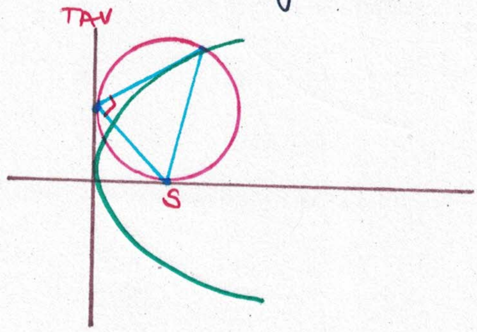


$\angle TSP = \angle TSQ$

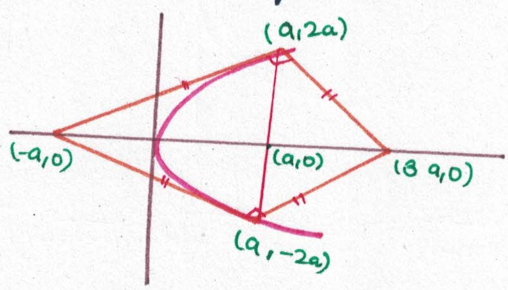
12 If we take focal chord as a diameter, then the circle will always touch the directrix.



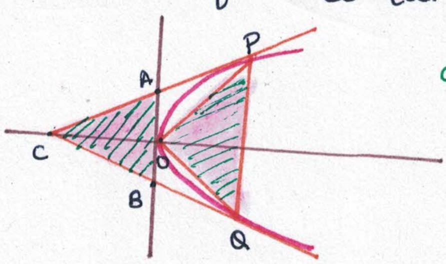
13 If we draw a circle considering focal length as diameter, then the circle will always touch TAV.



14 If we draw tangents and normals at the extremities of a latus rectum, it will form a square.



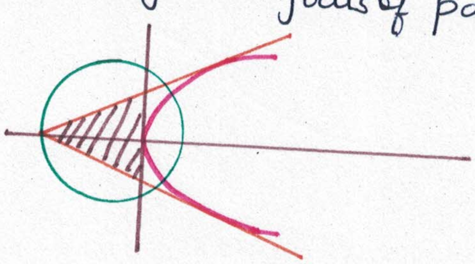
15 If we draw 3 tangents to the parabola $y^2 = 4ax$, then the area of the Δ formed by these 3 tangents is half of the area of the Δ formed by point of contact of these tangents.



area of higlighted Δ ACB

$= \frac{1}{2}$ area of Δ POQ

16 The circumcircle of the ΔABC shown in the given diagram will always pass through the focus of parabola.



17 If 3 tangents are drawn at pt $A(t_1)$, $B(t_2)$ and $C(t_3)$ on the parabola $y^2 = 4ax$ then the orthocenter of Δ formed by these 3 tangents always lies on directrix at orthocenter -

$[-a, a(t_1 + t_2 + t_3 + t_1 \cdot t_2 + t_2 \cdot t_3 + t_3 t_1)]$

18 Reflection Property

If any ray is incident on parabola parallel to AOP, then after reflection it will pass through focus and vice versa.